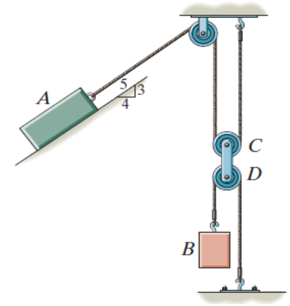
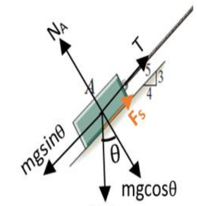
DYNAMICS

PROBLEM.15: At the instant shown the 100-lb block A is moving down the plane at 5 ft/s while being attached to the 50-lb block B. If the coefficient of kinetic friction between the block and the incline is μ_k = 0.2, determine
the acceleration of A and the distance A slides before it stops. Neglect the mass of the pulleys and cables.





Solution

Block A:

(+↗) $\sum F_x = ma_x$; $m_A g \sin\theta - T_A - \mu_k N_A = m_A a_A$

(+↖) $\sum F_y = 0$; $N_A - mg\cos\theta = 0 \rightarrow N_A = mg\cos\theta = 100(4/5) = 80$ lb

$100(0.6) - 0.2(80) - T = (\frac{50}{32.2})a_A \rightarrow 44 - T = (100/32.2)\, a_A$

$\therefore\ 44 - T = 3.105\, a_A$ (1)

Block B:

(+↑) $\sum F_y = ma_y$; $T_B - mg = ma_B \rightarrow T_B - 50 = (50/32.2)a_B$

$\therefore\ T_B - 50 = 1.552a_B$ (2)

$T_A = T_B$ and $a_A = a_B$ and byEq 2 multiplying by 2

$2T_B - 100 = 3.105a_B$ (3)

From Eqs 1, 3 we get

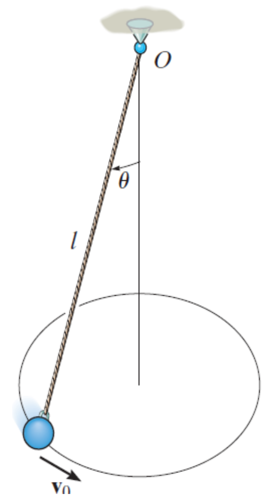
$\therefore 2T - 100 = 44 - T \rightarrow 3T = 144 \rightarrow T = 48$ N

By substituting t=48 in Eq 1 or 3

$48 - 50 = 1.552a_B \rightarrow\ a = -2/1.552 = -1.28$ ft/s

$v^2 = v_0^2 + 2aS \rightarrow\ 0 = 5^2 + 2(-1.28)(S)$

S = 25/2.56= 9.76 ft



PROBLEM.16: the ball has a mass m and is attached to the cord of length l. the cord is tied at the top to a swivel and the ball is given velocity. Show that the angle, which the cord makes with the vertical as the ball, travels around the circular path must satisfy the equation. Neglect air resistance and the size of the ball. Prove that (tanθ sinθ) = v_0^2/gl

SOLUTION

$(\overset{\rightarrow}{+})$ $\sum F_n = ma_n$; $T\sin\theta = m\,(v_0^2/r)$ 1

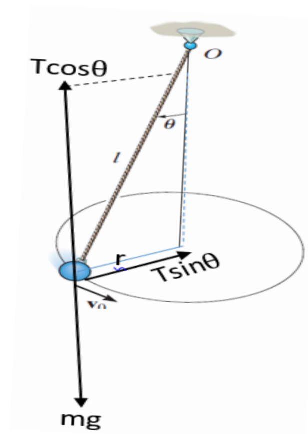
$(+\uparrow)$ $\sum F_y = 0$; $T\cos\theta - mg = 0 \rightarrow T\cos\theta = mg$ 2

By dividing Eq 1 on 2 then we get

$$\frac{T\sin\theta}{T\cos\theta} = \frac{m\frac{v^2}{r}}{mg} \rightarrow \tan\theta = v_0^2/gr \qquad 3$$

$\sin\theta = \frac{r}{l}$; r= l sinθ, by substituting r value in Eq 3 we obtain

$\tan\theta = v_0^2/gr \rightarrow \tan\theta.\ \sin\theta = v_0^2/gl$

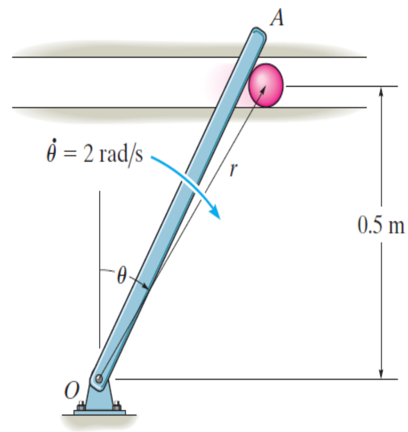


Useful trigonometric relation

Angle (θ)	0	30	37	45	53	60	90	180
Sinθ	0	1/2	3/5	$1/\sqrt{2}$	4/5	$\sqrt{3}/2$	1	0
Cosθ	1	$\sqrt{3}/2$	4/5	$1/\sqrt{2}$	3/5	1/2	0	-1
tanθ	0	$1/\sqrt{3}$	3/4	1	4/3	$\sqrt{3}$	∞	0
Secθ	1	$2/\sqrt{3}$	5/4	$\sqrt{2}$	5/3	$2/\sqrt{3}$	∞	-1
Cscθ	∞	2	5/3	$\sqrt{2}$	5/4	$2/\sqrt{3}$	1	∞
Cotθ	∞	$\sqrt{3}$	4/3	1	3/4	$1/\sqrt{3}$	0	∞

DYNAMICS

PROBLEM.17: Along the smooth horizontal, slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = 30°$. the rod is rotating with a constant angular velocity $\dot{\theta} = 2$ rad/s. Assume the particle contacts only one side of the slot at any instant.



Solution

$\cos\theta = \frac{0.5}{r} \rightarrow r = 0.5 \times \frac{1}{\cos\theta} = 0.5 \sec\theta$ (1)

$\dot{r} = 0.5 \sec\theta \tan\theta . \dot{\theta}$

$\ddot{r} = 0.5 [\sec\theta . \sec^2\theta . \dot{\theta} + \sec\theta \tan^2\theta . \dot{\theta}] . \dot{\theta} + 0.5 \sec\theta \tan\theta . \ddot{\theta}$ (2)

$\ddot{r} = 0.5 [\sec^3\theta + \sec\theta \tan^2\theta] . \dot{\theta}^2 + \sec\theta \tan\theta . \ddot{\theta}$ (3)

When $\theta = 30°$, $\dot{\theta} = 2$ rad/s, $\ddot{\theta} = 0$; (because the particle has a constant angular velocity)

$r = 0.5 \sec\theta = 0.5 (\sec 30) = 0.557$ m

$\dot{r} = 0.5 \sec\theta \tan\theta . \dot{\theta} = 0.5 \times [\sec 30 \times \tan 30 \times (2)] = 0.666$ m/s

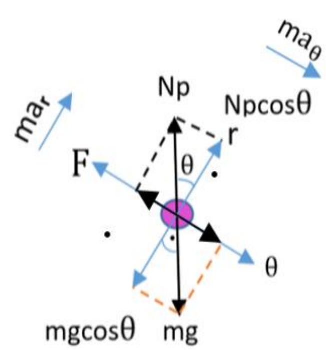
$\ddot{r} = 0.5 [\sec^3\theta + \sec\theta \tan^2\theta] . \dot{\theta}^2 + \sec\theta \tan\theta . \ddot{\theta}$

$\ddot{r} = 0.5 [\sec^3 (30) + \sec (30) \tan^2 (30)] . (2)^2 + \sec (30) \tan(30) \times 0] = 3.849$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = 1.5396$ m/s^2, $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.667$ m/s^2

(+↗) $\sum F_r = m a_r \Rightarrow N_P \cos\theta - mg \cos\theta = m a_r$

$0.86 N_P - 0.5 (9.81)(0.86) = 0.5(1.5396) \rightarrow N_P = 5.8$ N



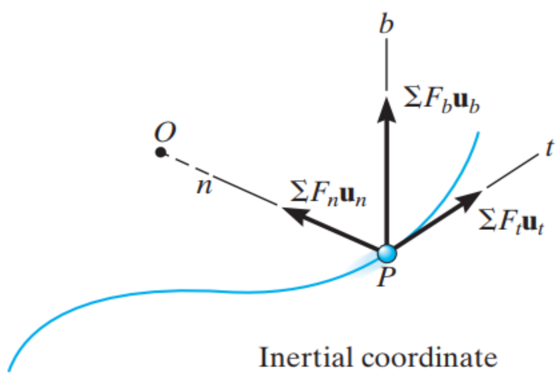
(+↘) $\Sigma F_\theta = m a_\theta$; $F - N_P \sin\theta + mg \sin\theta = m a_\theta$

$F - 5.8 (0.5) + 0.5(9.81) (0.5) = 0.5(2.667)$

$F = 1.781$ N

1.5 Equations of Motion: Normal and Tangential Coordinates.

When a particle moves along a curved path, which is known, the equation of motion for the particle may be written in the tangential, normal, and binormal directions, Fig. note that there is no motion of the particle in the binormal directions, since the particle is constrained to move along the path. We have



Inertial coordinate system

$$\sum F = ma$$

$$\sum F_t \theta_t + \sum F_n \theta_n + \sum F_b \theta_b = \sum m a_t + \sum m a_n$$

This equation is satisfied provided

$$\left.\begin{aligned} F_t &= m a_t \\ F_n &= m a_n \\ F_b &= 0 \end{aligned}\right\} \qquad (1\text{–}8)$$

Recall that $a_t = (dv/dt)$ represents the time rate of change in the magnitude of velocity. So if F_t acts in the direction of motion, the particle's speed will increase, whereas if it acts in the opposite direction, the particle will slow down. Likewise,

$a_n = (v^2/r)$ represents the time rate of change in the velocity's direction. It is caused by $\sum F_n$, which always acts in the positive n direction, i.e., toward the path's center of curvature. For this reason, it is often refer to as the centripetal force.

Procedure for Analysis

When a problem involves the motion of a particle along a known curved path, normal and tangential coordinates should be consider for the analysis since the acceleration components can be readily formulate. The method for applying the equations of motion, which relate the forces to the acceleration, has been out line in the procedure given in Sec. 1-4. Specifically, for t, n, b coordinates it may be state as follows:

Free-Body Diagram.

- Establish the inertial t, n, b coordinate system at the particle and draw the particle's free-body diagram FBD.
- The particle's normal acceleration $\mathbf{a}_n$ always acts in the positive n direction.
- If the tangential acceleration $\mathbf{a}_t$ is unknown, assume it acts in the Positive t direction.
- There is no acceleration in the b direction.
- Identify the unknowns in the problem.

Equations of Motion.

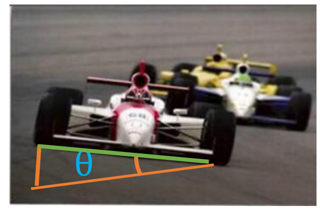
- Apply the equations of motion, Eq. 1–8.

Kinematics.

- Formulate the tangential and normal components of acceleration; i.e., $a_t = dv/dt$ or $a_t = v\,dv/ds$ and $a_n = v^2/\rho$.
- If the path is defined as $y = f(x)$, the radius of curvature at the point where the particle is located can be obtained from

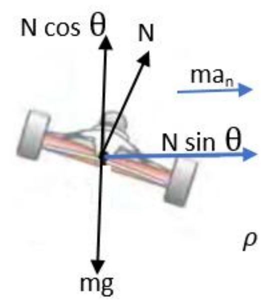
$$\rho = [1 + (\frac{dy}{dx})^2]^{3/2} / \left| \frac{d^2y}{dx^2} \right|$$

PROBLEM.18: Determine the banking angle θ for the racetrack so that the wheels of the racing cars shown in Fig. 1-12 will not have to depend upon friction to prevent any car from sliding up or down the track. Assume the cars have negligible size, a mass m, and travel around the curve of radius r with a constant speed v.



SOLUTION

Before looking at the following solution, give some thought as to why it should be solved using t, n, b coordinates.



Free-Body Diagram. As shown in Fig. 1–12b, and as stated in the problem, no frictional force acts on the car. Here **N** represents the resultant of the ground on all four wheels. Since can be calculated, the unknowns are N and θ.

Equations of Motion. Using the n, b axes shown,

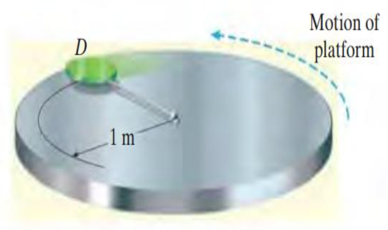
$(\overset{\rightarrow}{+})$ $\sum F_n = ma_n;\quad N\sin\theta = m\frac{v^2}{\rho}$ (1)

$(+\uparrow)$ $\sum F_b = 0;\quad N\cos\theta = mg$ (2)

Eliminating N and m from these equations by dividing Eq. 1 by Eq. 2, we obtain

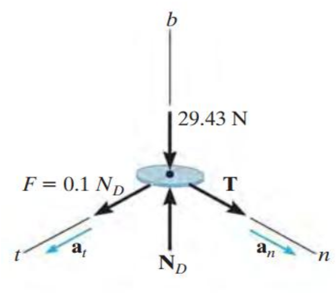
$\tan\theta = \frac{v^2}{\rho g};\quad \theta = \tan^{-1}\left(\frac{v^2}{\rho g}\right)$

PROBLEM.19: The 3-kg disk D is attached to the end of a cord as shown in Fig. 1-13a. The other end of the cord is attached to a ball-and-socket joint located at the center of a platform. If the platform rotates rapidly, and the disk is placed on it and released from rest as shown, determine the time it takes for the disk to reach a speed great enough to break the cord. The maximum tension the cord can sustainable is 100 N, and the coefficient of kinetic friction between the disk and the platform is $\mu_k = 0.1$



SOLUTION

Free-Body Diagram. The frictional force has a magnitude $F = \mu_k N = 0.1N$ and a sense of direction that opposes the relative motion of the disk with respect to the platform. It is this force that gives the disk a tangential component of acceleration causing v to increase, thereby causing T to increase until it reaches 100 N. The weight of the disk is W = 3(9.81) = 29.43 N. Since an can be related to v, the unknowns are ND, at, and v.



(b)

(+↙) $\sum F_n = ma_n;\ T = m(\frac{v^2}{\rho}) \rightarrow T = 3(\frac{v^2}{\rho})$ (1)

(+↘) $\sum F_t = ma_t;\ F_K = ma_t \rightarrow \mu_k N = 3\,a_t$ (2)

(+↑) $\sum F_b = 0;\ N - mg = 0$ (3)

Setting T = 100 N, Eq. 1 can be solved for the critical speed v_{cr} of the disk needed to break the cord. Solving all the equations,

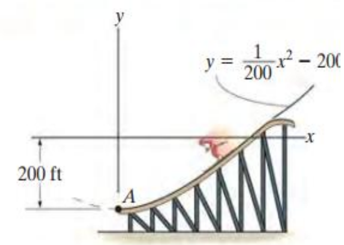
From Eqs 3, 2,1 can be obtain

$N = mg = 3(9.81) = 29.43$ N , $a_t = 0.1(29.43)/3 = 0.981\ m/s^2$,

And $100 = 3(\frac{v_{cr}^2}{1}) \rightarrow\ v_{cr}^2 = v_{cr} = 5.77$ m/s

Kinematics. Since *at* is *constant*, the time needed to break the cord is

$v_{cr} = v_0 + a_t t$, 5.77 = 0 + (0.981)t; t = 5.89 s

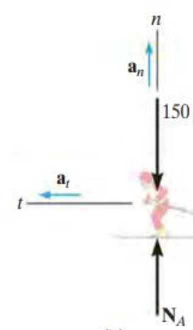


(a)

PROBLEM.20: Design of the ski jump shown in the photo requires knowing the type of forces that will be exerted on the skier and her approximate trajectory. If in this case the jump can be approximat--ed by the parabola shown in Fig. 1-14a, determine the normal force on the 150-lb skier the instant she arrives at the end of the jump, point A, where her velocity is 65 ft/s. In addition, what is her acceleration at this point?

SOLUTION

Why consider using n, t coordinates to solve this problem?



(b)

Free-Body Diagram. Since dy/dx = x/100, x=0, the slope at A is horizontal. The free-body diagram of the skier when it is at A is shown in Fig. 1–14b. Since the path is curved, there are two components of acceleration, $\mathbf{a}_n$ and $\mathbf{a}_t$. thereby we can be calculate , the unknowns are a_t and N_A.

(+↑) $\sum F_n = ma_n$, $N_A - mg = m\left(\frac{v^2}{\rho}\right) \Rightarrow$

$$N_A - 150 = (150/32.2)\left(\frac{65^2}{\rho}\right) \quad (1)$$

($\overset{\leftarrow}{+}$) $\sum F_t = ma_t$, $0 = (150/32.2)\, a_t \Rightarrow a_t = 0$ (2)

The radius of curvature ρ for the path must be determined

A(0, -200 ft) . Here $y = \frac{1}{200}x^2 - 200 \Rightarrow \frac{dy}{dx} = \frac{1}{100}x$ and $\frac{d^2y}{dx^2} = \frac{1}{100}$, so that at x = 0

$$\rho = [1 + (\tfrac{dy}{dx})^2]^{3/2} / \left|\tfrac{d^2y}{dx^2}\right|_{x=0} \Rightarrow \rho = [1 + (\tfrac{1}{100} \times 0)^2]^{3/2} / \left|\tfrac{1}{100}\right| = 100 \text{ ft}$$

by Substituting this into Eq. 1 and solving for N_A, we obtain $N_A = 347$ lb

Kinematics. From Eq. 2 $a_n = \left(\frac{v^2}{\rho}\right) \Rightarrow \left(\frac{65^2}{100}\right) = 42.2 \text{ ft/s}^2$,

$a_A = a_n = 42.2 \text{ ft/s}^2 \uparrow$