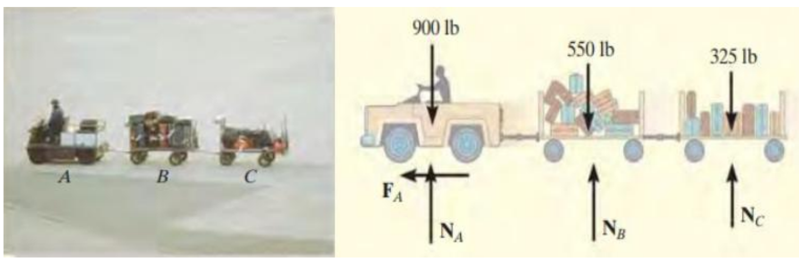
DYNAMICS

PROBLEM.3: The baggage truck A shown in the photo has a weight of 900 lb and tows a 550-lb cart B and a 325-lb cart C. For a short time the driving frictional force developed at the wheels of the truck is F_A = (40t) lb, where t is in seconds. If the truck starts from rest, determine its speed in 2s. also, what is the horizontal force acting on the coupling between the truck and cart B at this instant? Neglect the size of the truck and carts.



SOLUTION

Free-Body Diagram. As shown in Fig.1-8, the frictional driving force gives both the truck and carts an acceleration. Here we have considered all three vehicles as a single system

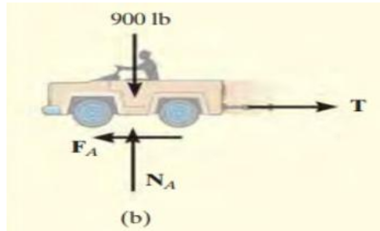
Equation of Motion. Only motion in the horizontal direction has to be considered.

$(\overset{\leftarrow}{+})$ $\sum F_x = ma_x$; 40t = a (900 + 550 + 325)/32.2 then a = 0.7256t

Kinematics. Since the acceleration is a function of time, the velocity of the truck is obtained using a= dv/dt with the initial condition that v0 = 0 at t = 0. We have

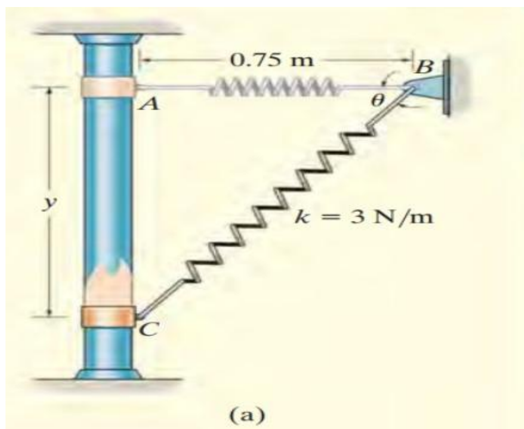
dv =a dt ; $\int_0^{v} dv = \int_0^{25} 0.7256t\, dt \rightarrow$ v= $0.3628t^2]_0^{25}$ = 1.45 ft/s

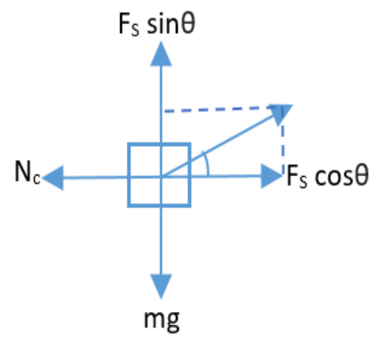
Free-Body Diagram. In order to determine the force between the truck and cart B, we will consider a free-body diagram of the truck so that we can "expose" the coupling force **T** as external to the free-body diagram, Fig. 1-8b



Equation of Motion. When t = 2 s, then

$(\overset{\leftarrow}{+})\ \sum F_x = ma_x$; $40t - T = a\,(900)/32.2$ but we have t=2, a=0.7256t then T=39,4 lb





Homework: Try to obtain

this same result by

considering a free-body diagram of carts B and C as a single system.

PROBLEM.4: A smooth 2-kg collar, shown in Fig. 1–9a, is attached to a spring having a stiffness k = 3 N/m and an upstretched length of 0.75 m. If the collar is released from rest at A, determine its acceleration and the normal force of the rod on the collar at the instant y = 1 m.

SOLUTION

Free-Body Diagram.

The free-body diagram of the collar when it is located at the arbitrary position y is shown in Fig. 1–9b. Furthermore, the collar is assumed to be accelerating so that "a" acts downward in the positive y direction. There are four unknowns, namely, N_C, Fs, a, and θ.

s =1.25 - 0.75 = 0.5 m since

Fs = ks = 3(0.5)= 1.5 N (1)

Equations of Motion.

$(\overset{\leftarrow}{+})$ $\sum Fx = max$; ∴ -NC + F_{SP} cos θ = 0 (2)

$(+\downarrow)$ $\sum Fy = may$: ∴ mg- F_{SP}sin θ = ma (3)

From Eq. 2 it is seen that the acceleration depends on the magnitude and direction of the spring force. Solution for NC and a is possible once F_{SP} and u are known. The magnitude of the spring force is a function of the stretch s of the spring; Fs = ks. Here the unstretched length is AB = 0.75 m, Fig. 1–9a; therefore,

S = CB - AB =L-L_0 =$\sqrt{y^2+0.75^2}$ -0.75 = $\sqrt{1^2+0.75^2}$ -0.75

from Eq. 3. mg- F_{SP}sin θ = ma , 2(9.81) - 1.5 (0.8) = 2a

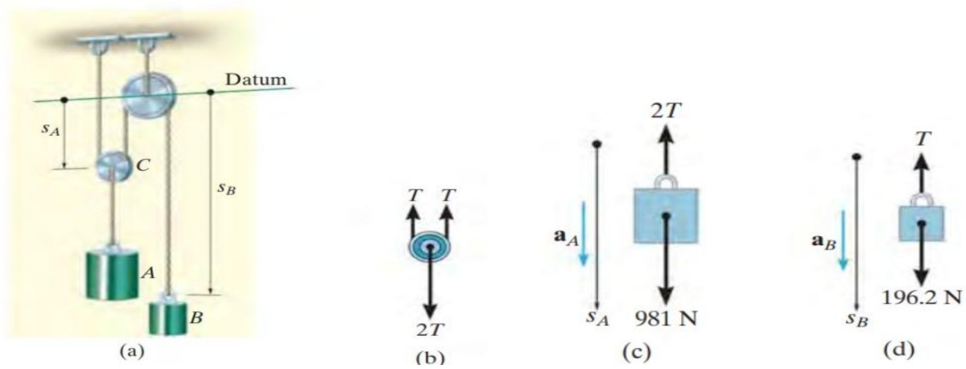
then we get a = (19.96-0.4)/2= 9.3

From Fig. 1–9a, tan θ = 1/0.75 = 4/3; θ=53^0Substituting θ in Eq. 1,

$N_C = F_{SP}\cos\theta = 1.5\times 0.6 = 0.9$ N

NOTE: This is not a case of constant acceleration, since the spring force changes both its magnitude and direction as the collar moves downward.

PROBLEM.5: the 100-kg block A shown in Fig. 1-10a is released from rest. If the masses of the pulleys and the cord are neglected, determine the velocity of the 20-kg block B in 2 s



SOLUTION

Free-Body Diagrams. Since the mass of the pulleys is neglected, then for pulley C, ma = 0 and we can apply Fy = 0, as shown in Fig. 1–10b. The freebody diagrams for blocks A and B are show in Fig. 1–10c and d, respectively. Notice that for A to remain stationary T = 490.5 N, whereas for B to remain static T = 196.2 N. Hence A will move down while B moves up. Although this is the case, we will assume both blocks accelerate downward, in the direction of +sA and +sB. The three unknowns are T, aA, and. , aB

Equations of Motion for Block A,

(+↓) $\sum F y = m a_A$; $mg - 2T = m\, a_A$ 1

For block B

(+↓) $\sum F = ma_B$; $mg - T = m a_B$ 2

Kinematics. The necessary third equation is obtained by relating aA to aB using a dependent motion analysis, discussed in Sec. 1.9a. The coordinates sA and sB in Fig. 1–10a measure the positions of A and B from the fixed datum. It is seen that

$$2S_A + S_B = L$$

where L is constant and represents the total vertical length of cord. Differentiating this expression twice with respect to time yields

$$2a_A = -a_B \qquad 3$$

Notice that when writing Eqs. 1 to 3, the positive direction was always assumed downward. It is very important to be consistent in this assumption since we are seeking a simultaneous solution of equations. The results are

$$T = 327.0 \text{ N}, \; a_A = 3.27 \text{ m/s2}, \; a_B = -6.54 \text{ m/s}^2$$

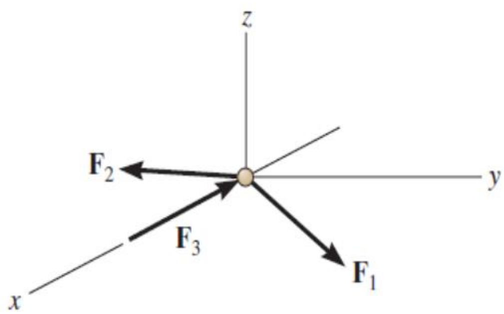
Hence when block A accelerates downward, block B accelerates upward as expected. Since aB is constant, the velocity of block B in 2 s is thus

$$v = v_0 + a_B t = 0 + (-6.54)(2) = -13.1 \text{ m/s}$$

The negative sign indicates that block B is moving upward

VARIOUS PROBLEMS

PROBLEM.6: The 6 1b particle is subjected to the action of its weight and forces F_1=2i + 6j - 2tk lb, $F_2 = t^2$i - 4tj - k lb, and F_3= - 2ti-6k lb, and where t is in seconds. Find the distance the particale is from the origin 2s after being released from rest.



SOLUTION

Equations: $\sum F = ma;\ \sum Fxi + \sum Fyj + \sum Fzk = m(a_x i + a_y j + a_z k)$

$\sum F=ma;\ (2i + 6j - 2tk) + (t^2 i - 4tj - k) - (2ti - 6k) = (6\ 32.2)(axi + ayj + azk)$

Equating component

$(6/32.2)a_x = (t^2-2t+2)$, $(6/32.2)a_y = (6-4t)$, $(6/32.2)a_z = (-2t-7)$

Since dv= adt, integrating from s=0, t=0 yields

$(6/32.2)v_x = (t^3/3 - t^2+2t)$, $(6/32.2)v_y = (6t-2t^2)$, $(6/32.2)v_z = (-t^2-7t)$

Since ds= vdt, integrating from s=0, t=0 yields

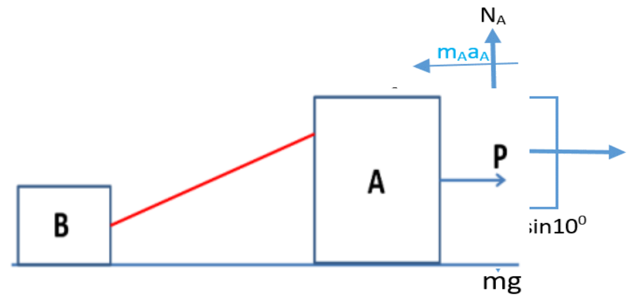
$(6/32.2)S_x = (t^4/12 - t^3/3+t^2)$, $(6/32.2)S_y = (3t^2 - 2t^3/3)$, $(6/32.2)sz = (-t^3/3 - 7t^2/2)$

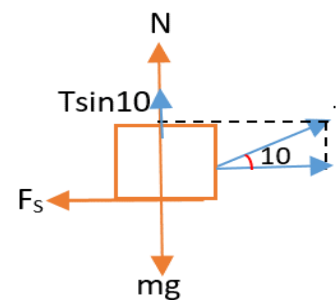
When t= 2s, by substitute in the last Eqs, then,

$S_x=14.31$ ft , $S_y=35.78$ ft, $S_z=-89.44$ ft

$S=\sqrt{S_x^{\ 2} + S_y^{\ 2} + S_z^{\ 2}} = \sqrt{14.31^2 + 35.78^2 + (-89.44)^2} = 97.4$ ft

PROBLEM.2: a horizontal force P=70 N is exerted on mass A=16kg as shown in Fig. below. The coefficient of friction between mass A and the horizontal plane is 0.25. B has amass of 4 kg and the coefficient of friction between it and the plane is 0.5. the cord between the two masses makes an angle of 10° with the horizontal.What is the tension in the cord.





Solution

For body A

$\sum Fy = 0$, a_{yA} $\Rightarrow$ $N_A - mg - T\sin\theta = 0$,

$N_A = 156.96 + T\sin 10° = 156.96 + 0.173\,T$

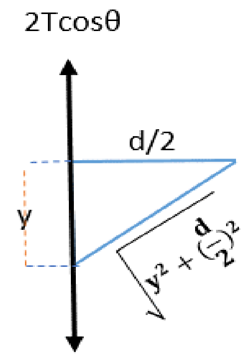
$F_f = \mu_f N_A = 0.25\,(156.96 + 0.173\,T)$

$\sum Fx = m_A a_x \longrightarrow 70 - F_f - T\cos 10° = 16\,a_x$

$70 - 0.25\,(156.96 + T\sin 10°) - T.\cos 10° = 16 a_A$

$a_{xA} = 1.923 - 0.0643T$ (1)

For body B



$\sum F_y = 0$, $N_B = 4 \times 9.81 - T \sin 10^o$

$N_B = 39.24 - 0.173\,T$

$\sum F_x = m_B\, a$, $T \cos 10^o - F_f = m a_B$

$T \cos 10^o - \mu_B (39.24 - 0.173\, T) = 4 a_B$

$0.984\, T - 0.5\,(39.24 - 0.173\, T) = 4 a_B$

$0.984\, T - 19.62 + 0.065 T = 4 a_B$; $1.049T - 19.62 = 4 a_B$

$a_B = 0.262\, T - 4.905$ (2)

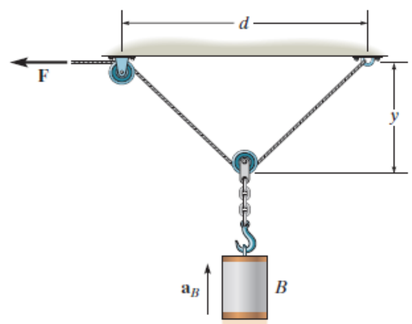
$a_A = a_B$, ∴ $1.923 - 0.0643T = 0.262\, T - 4.905$, $0.326\, T = 6.828$ then $T = 20.9$ N

PROBLEM.8: Cylinder B has a mass m and is hoisted using the cord and pulley system shown. Determine the magnitude of force **T** as a function of the block's vertical position y so that when **T** is applied the block rises with a constant acceleration $\mathbf{a}_B$. Neglect the mass of the cord and pulleys.

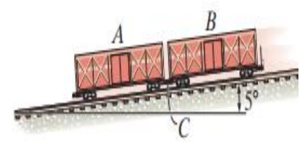
(+↑) $\sum F_y = m a_y$; $2T \cos\theta - mg = m a_B$

where $\cos\theta = \dfrac{y}{\sqrt{y^2 + (\frac{d}{2})^2}}$

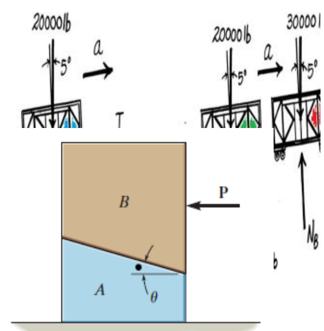
$$\frac{4Ty}{\sqrt{4y^2 + d^2}} = m\,(a_B + g) \rightarrow T = \frac{m\,(a_B + g)\sqrt{4y^2 + d^2}}{4y}$$



PROBLEM.9: the two boxcars A and B have a weight of A 20000 lb and B 30000 lb respectively. If they are freely coasting down the incline when the brakes are applied to all the wheels of car A, determine the force in the coupling C between the two cars. The coefficient of kinetic friction between the wheels of A and the tracks is μ_k=0.5, the wheels of car B are free to roll. Neglect their mass in the calculation. Suggestion: Solve the problem by representing single resultant normal forces acting on A and B, respectively



For car A:



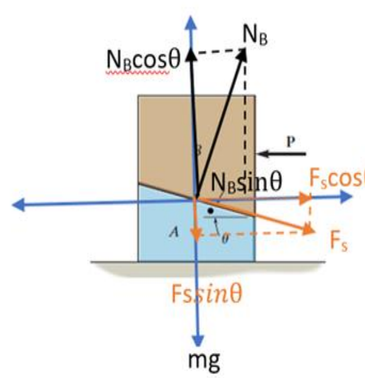
(+↖) $\sum F_y = 0$; $N_A - mg_A \cos\theta = 0$, $N_A = mg \cos 5°$

N_A = 20000 (0.996) = 19 923.89 lb

(+↗) $\sum F_x = ma_x$; $- mg_A \sin\theta - T + F_k - = ma_A$, $F_K = \mu_k N_A$

-20000 sin5°-T+ 0.5(19 923.89) = (20000/32.2) a_A

8218.83 – T= 621.11 a_A (1)



For both car A,B; $m_{AB}=m_A+m_B$ = 50000 Ib

(+↗) $\sum F_x = m_{AB}a_{AB}$; $F_k - mg_{AB} \sin 5° = (m_{AB}/32.2)\, a_{AB}$

0.5(19 923.89) -50000 sin 5°= (50000/32.2) a_{AB}

5604.16=1552.8 a_{AB} (2)

By solving Eqs. 1, 2 then we get

$a_A = a_{AB}$ =3.61 ft/s^2 and T= 5.98 K1b

PROBLEM.10: Block A and B each have a mass m. Determine the largest horizontal force **P** which can be applied to B so that it will not slide on A. Also, what is the corresponding acceleration? The coefficient of static friction between A and B is μ_s. Neglect any friction between A and the horizontal surface.

Solution

Equations of Motion. Since block, B is required to be on the verge to slide on A, $F_f = \mu_s N_B$. Referring to the FBD of block B shown in Fig. a,

$(+\uparrow)$ $\sum F_y = ma_y$; $N_B \cos\theta - \mu s N_B \sin\theta - mg = m(0)$

$$N_B = \left(\frac{mg}{\cos\theta - \mu_s \sin\theta}\right) \qquad (1)$$

$(\overset{\leftarrow}{+})$ $\sum F_x = ma_x$; $P - N_B \sin\theta - \mu s N_B \cos\theta = ma$

$$P - N_B (\sin\theta + \mu_s \cos\theta) = ma \qquad (2)$$

Substitute Eq. (1) into (2), then we get

$$P - \left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right) mg = ma \qquad (3)$$

Referring to the FBD of blocks A and B shown in Fig. b

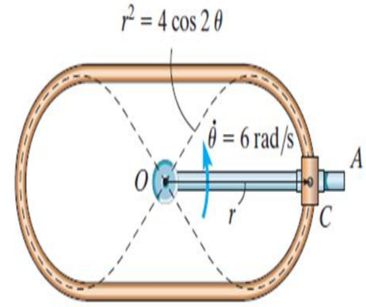
$(\overset{\leftarrow}{+})$ $\sum F_x = ma_x$; $P = 2ma$ (4)

Solving Eqs. (2) into (3),

$$2ma - \left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right) m.g = ma, \text{ from this}$$

$$m.a = \left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right) m.g \rightarrow a = \left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right) g \text{ and } p = 2\left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right) mg$$

PROBLEM.11: the automobile has a speed of 80 ft/s at point A and an acceleration **a** having a magnitude of 10 ft/s^2, acting in the direction shown. Determine the radius of curvature of the path at point A and the tangential component of acceleration.



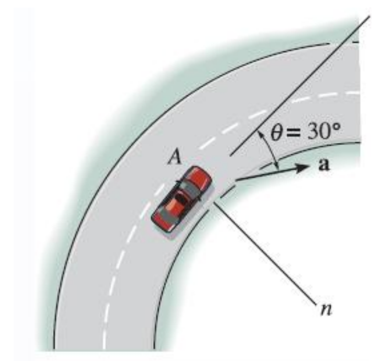
SOLUTION

Acceleration: The tangential acceleration is

$a_t = a\cos\theta = 10\cos 30° = 8.66$ ft/s^2

and the normal acceleration is $a_n = a\sin 30° = 10\sin 30° =$ 5.00 ft/s^2.

Applying $a_n = \frac{v^2}{\rho}$ the we have $\rho = \frac{v^2}{a_n} = \frac{80^2}{5} = 1280$ ft



PROBLEM.12: the satellite *S* travels around the earth in a circular path with a constant speed of 20 Mm/h. If the acceleration is 2.5 m/s^2, determine the altitude h. Assume the earth's diameter to be 12 713 km.

SOLUTION

$v = 20\times10^3 \times \frac{5}{18} = 5.56 \times 10^3$ m/s

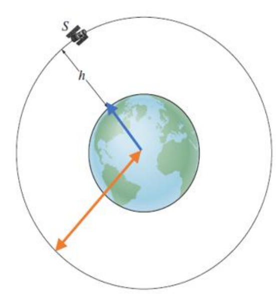
$a_t = \frac{dv}{dt} = 0;\quad a = a_n = 2.5 = \frac{v^2}{R} \to R = \frac{(5.56\times10^3)^2}{2.5} = 12.35$ Mm

The radius of the earth is $r = 12\,713/2 = 6.3565$ Mm.

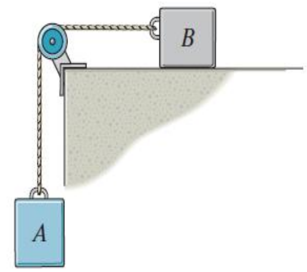
Hence $h = R - r = 12.35\times10^6 - 6.3565\times10^6 = 5.99$ Mm

NOTE Km/h = (5/18) m/s

PR0BLEM.13: A double collar C is pin connected together such that one collar slides over a fixed rod and the other slides over a rotating rod. If the geometry of the fixed rod for a short distance can be defined by a lemniscate, $r^2 = 4\cos 2\theta)$ ft^2, determine the collar's radial and transverse components of velocity and acceleration at the instant $\theta = 0°$ as shown. Rod OA is rotating at a constant rate of $\dot{\theta} = 6$ rad/s.



SOLUTION



$r^2 = 4\cos 2\theta$ (1)

$2r\dot{r} = -4\sin(2\theta)(2\dot{\theta})$;

$r\dot{r} = -4\sin(2\theta)(\dot{\theta})$ (2)

$r\ddot{r} + \dot{r}^2 = -4\sin 2\theta\,\ddot{\theta} - 8\cos 2\theta.\,\dot{\theta}^2$ (3)

When $\theta = 0$, $\dot{\theta} = 6$, $\ddot{\theta} = 0$

$r = \sqrt{4\cos 0} = 2$,

$2\dot{r} = -4\sin 2(0)(\dot{\theta}) = 0\,(6) = 0$

$2\ddot{r} = 0 - 4\sin 2(0)\,(0) - 8\cos 2(0)(6)^2 \rightarrow$ $\ddot{r} = -288/2 = -144$

$V_r = \dot{r} = 0$

$V_\theta = r\dot{\theta} = 2(6) = 12$ ft/s , $v = \sqrt{(V_r)^2 + (V_\theta)^2} = 12$ ft/s

$a_T = \ddot{r} - r\dot{\theta}^2 = -144 - 2(6)^2 = -216$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2(0) + 2(0)(6) = 0$, $a = \sqrt{a_T{}^2 + a_\theta{}^2} = -216$ ft/s^2

Remember

$V_r = \dot{r}$, $V_\theta = r\dot{\theta}$

$a_r = \ddot{r} - r\dot{\theta}^2$, $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

PROBLEM.14: Determine the acceleration of the blocks when the system is released. The coefficient of kinetic friction is μ_k, and the mass of each block is m. Neglect the mass of the pulleys and cord.

Solution

DYNAMICS

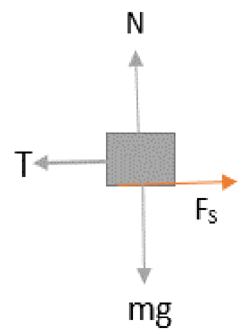
Free Body Diagram. Since the pulley is smooth, the tension is constant throughout the entire cord. Since block B is required to slide, $F_f = \mu_k N$. also, blocks A and B are attached together with inextensible cord, so $a_A = a_B = a$. The FBDs of blocks A and B are shown in Figs. a and b, respectively.

Equations of Motion. For block A, Fig. a,

$\sum F_X = 0$

$(+\downarrow)$ $\quad \sum F_y = ma_y;\ mg - T = ma \qquad (1)$

For block B, Fig. b,



$\sum F_y = ma_y;\ N - mg = 0$ then $N = mg \qquad (2)$

$(\overset{\leftarrow}{+})$ $\quad \sum Fx = ma_x;\ T - \mu_k\, mg = ma \qquad (3)$

Solving Eqs. 1,2 and 3 where $m_A = m_B = m$

$mg - T = T - \mu_k\, mg, \quad 2T = mg + \mu_k\, mg \rightarrow T = \frac{1}{2} mg\,(1 + \mu_k) \qquad (4)$

by substituting Eq. 4 in 3 can we get

$\frac{1}{2}\, mg\,(1 + \mu_k) - \mu_k\, mg = ma \rightarrow \frac{1}{2} g + \frac{1}{2} \mu_k g - \mu_k g a = a \rightarrow a = \frac{1}{2}(1 - \mu_k) g$